

Math 10B with Professor Stankova

Quiz 11; Tuesday, 4/16/2019

Section #206; Time: 9:30 AM

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Name: \_\_\_\_\_

Circle True or False or leave blank. (1 point for correct answer,  $-1$  for incorrect answer, 0 if left blank)

1. True **FALSE** The 99% confidence interval will be smaller than the 95% confidence interval.

**Solution:** With a smaller interval, we are less confident the true parameter will lie inside the range.

2. True **FALSE** To make our 95% confidence interval twice as small, we need twice the sample size.

**Solution:** Since the width falls off as  $1/\sqrt{n}$ , we would need 4 times the sample size.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (5 points) The number of threes made during an NBA game is Poisson distributed. Last Sunday, the number of threes made were 16, 15, 23, and 22. What is the 95% confidence interval for the number of 3's made during a game?

**Solution:** We have

$$\hat{\mu} = \bar{x} = \frac{16 + 15 + 23 + 22}{4} = \frac{76}{4} = 19.$$

Then because this is a Poisson distribution, we have  $\hat{\mu} = \hat{\lambda}$  so  $\hat{\lambda} = 19$  and  $\hat{\sigma} = \sqrt{\hat{\lambda}} = \sqrt{19}$ . So the 95% confidence interval is

$$\left(19 - \frac{2\sqrt{19}}{\sqrt{4}}, 19 + \frac{2\sqrt{19}}{\sqrt{4}}\right) = (19 - \sqrt{19}, 19 + \sqrt{19}).$$

- (b) (5 points) Using the numbers from the previous part, calculate the maximum likelihood estimate for the parameter  $\lambda$ .

**Solution:** The likelihood function is  $L(\lambda|16, 15, 23, 22) = P(16, 15, 23, 22|\lambda) = \frac{\lambda^{16}e^{-\lambda}}{16!} \cdot \frac{\lambda^{15}e^{-\lambda}}{15!} \cdot \frac{\lambda^{23}e^{-\lambda}}{23!} \cdot \frac{\lambda^{22}e^{-\lambda}}{22!} = \frac{\lambda^{76}e^{-4\lambda}}{16!15!23!22!}$ . Taking the derivative and setting it equal to 0 gives

$$\lambda^{76}(-4e^{-4\lambda}) + 76\lambda^{75}e^{-4\lambda} = \lambda^{75}e^{-4\lambda}(-4\lambda + 76) = 0.$$

Thus  $\lambda = 0$  or  $\lambda = \frac{76}{4} = 19$ . Since  $\lambda = 0$  doesn't make sense and 19 is a local maximum, we have  $\lambda = 19$ .